Problem Solving Without Ansibles:

An Introduction to Communication Complexity

Chris Grossack (they/them)

Ansibles in Science Fiction:

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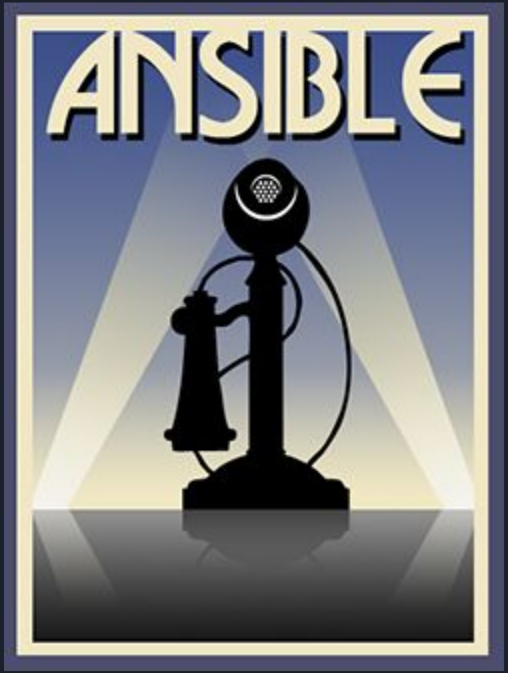

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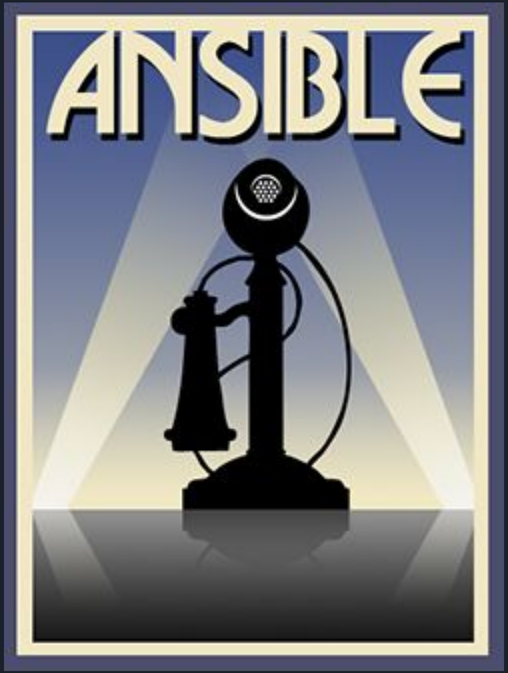

- Faster-than-light communication

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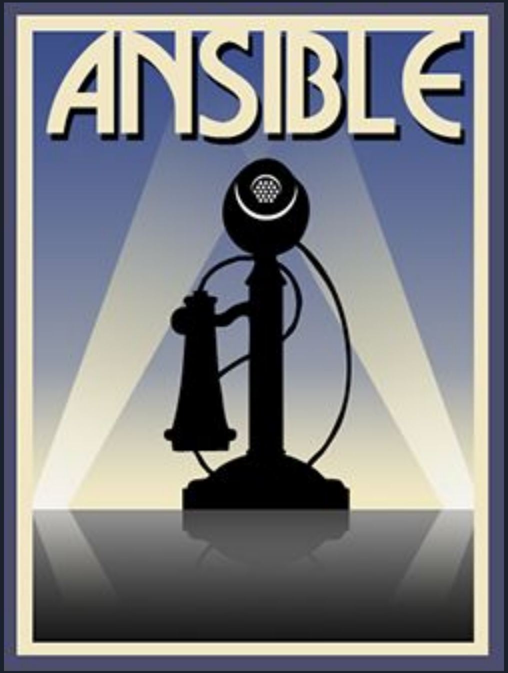

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- Allows colonies, etc. to speak in real time

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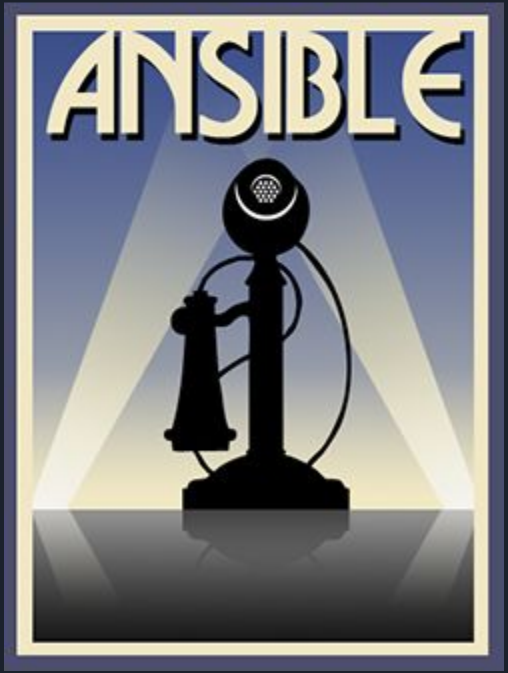

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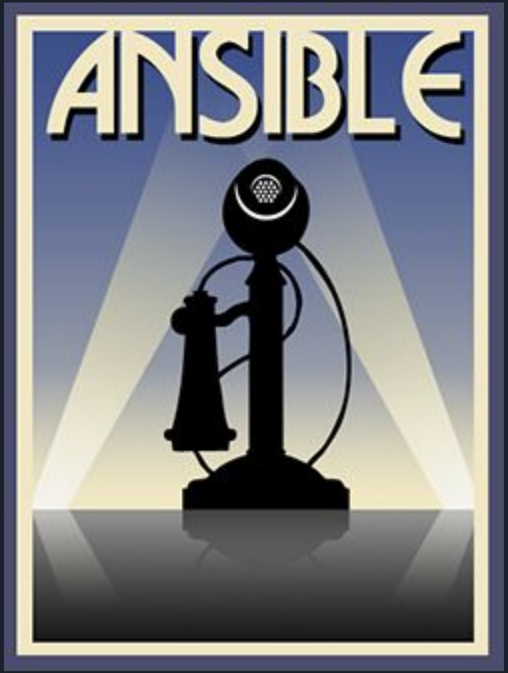

- Faster-than-light communication
- Allows colonies, etc. to speak in real time
- Generally helps the plot go brrrrrr
- Notably aphysical -- almost certainly impossible

In the real world...

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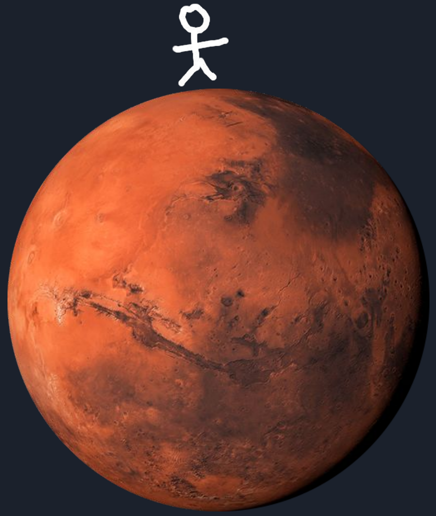
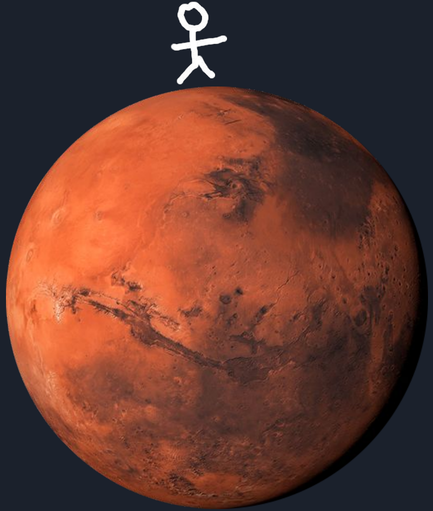


In the real world...

$a \in \{0,1\}^n$



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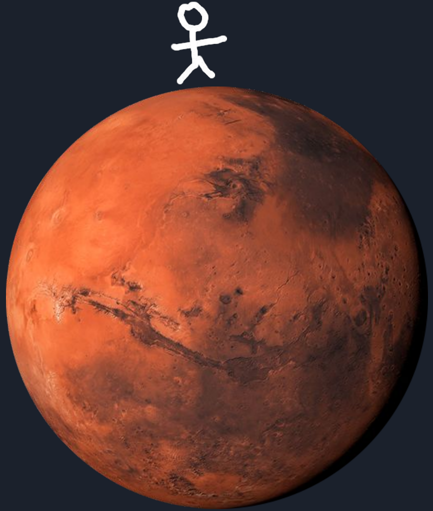

$f(a,b)$

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We only penalize **Communication** between Alyss and Bob

Some remarks:

- We never need $> n+1$ messages

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- In general, we want #messages $< \log(n)^k$ for some constant k

A simple example:

Write $D(f) \triangleq \min_{\text{protocol } \underline{P}} \max_{(a,b)}$ #messages sent using $\underline{P}$ to compute $f(a,b)$

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What is $D(Eq_n)$,

where $Eq_n(x,y) = \begin{cases} 1 & x = y \\ 0 & x \neq y \end{cases}$?

Theorem: Eq_n is maximally hard

(That is, $D(Eq_n) = n+1$)

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Idea:

$$\begin{array}{cc} & b \\ 2^n \; a & \begin{pmatrix} & \vdots & \\ \cdots & f(a,b) & \\ & & \end{pmatrix} \\ & 2^n \end{array}$$

Theorem: Eq_n is maximally hard

(That is, $D(Eq_n) = n+1$)

Idea:

$$
A \quad \begin{array}{c c}
\begin{array}{c} 00 \\ 01 \\ 10 \\ 11 \end{array} & \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\
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Let’s see a sample computation with some (suboptimal) protocol:

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3. Then Alyss sends her second bit (1) to Bob

    This restricts the region again, and now the only option is 0! So we know f(a,b) = 0

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- That is, a region of the form $A_0 \times B_0$, for $A_0 \subseteq A$ and $B_0 \subseteq B$

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- So! Any protocol correctly computing f must partition our matrix into combinatorial rectangles, each of which only has 0s or 1s inside it.
- In ~fancy~ lingo: Any protocol partitions the matrix into monochromatic rectangles

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This entry MUST be a 1x1 rectangle,
as any rectangle containing 2 rows
(resp. columns) must contain an off diagonal
entry.

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This means any protocol solving Eq_n has at least 2^n rectangles.

But each additional message sent splits a rectangle into 2 pieces.

So Eq_n requires at least $\log_2(2^n) = n$ many messages.

Ok... What if we only want to be correct with high probability?

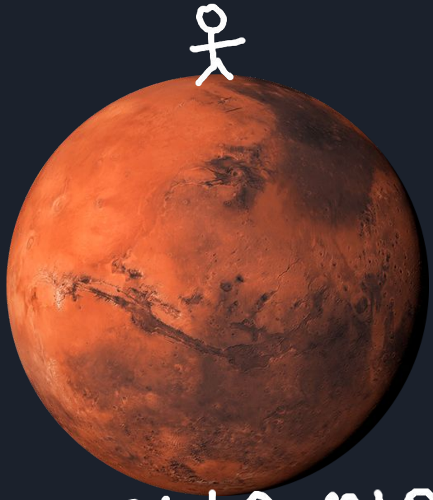
Theorem:
Alyss and Bob can communicate $O(\log n)$ bits, so that

1. If $a = b$, then we always correctly say “yes, they’re equal”

2. If $a \neq b$, then we incorrectly say “yes they’re equal” with probability $< 1/n$

(We say such an algorithm has “one-sided error”)

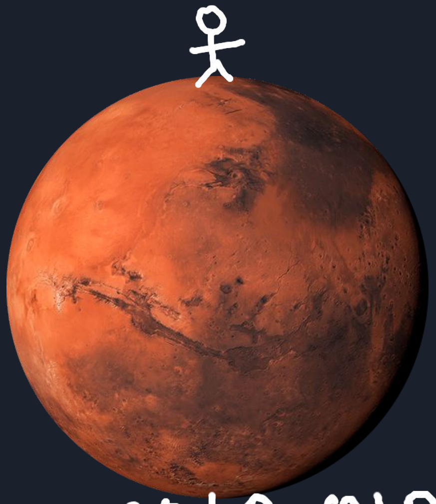
$a = 1010\ 0010$



$b = 0010\ 0101$

$$a = 1010\ 0010$$

$$p_a = 1x + 0x^2 + 1x^3 + 0x^4 + 0x^5 + 0x^6 + 1x^7 + 0x^8$$



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$n^2 < q < 2n^2$

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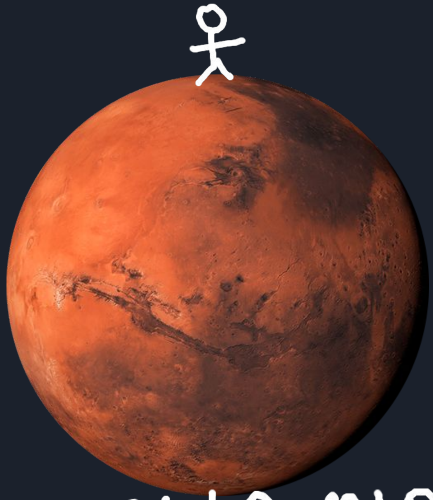

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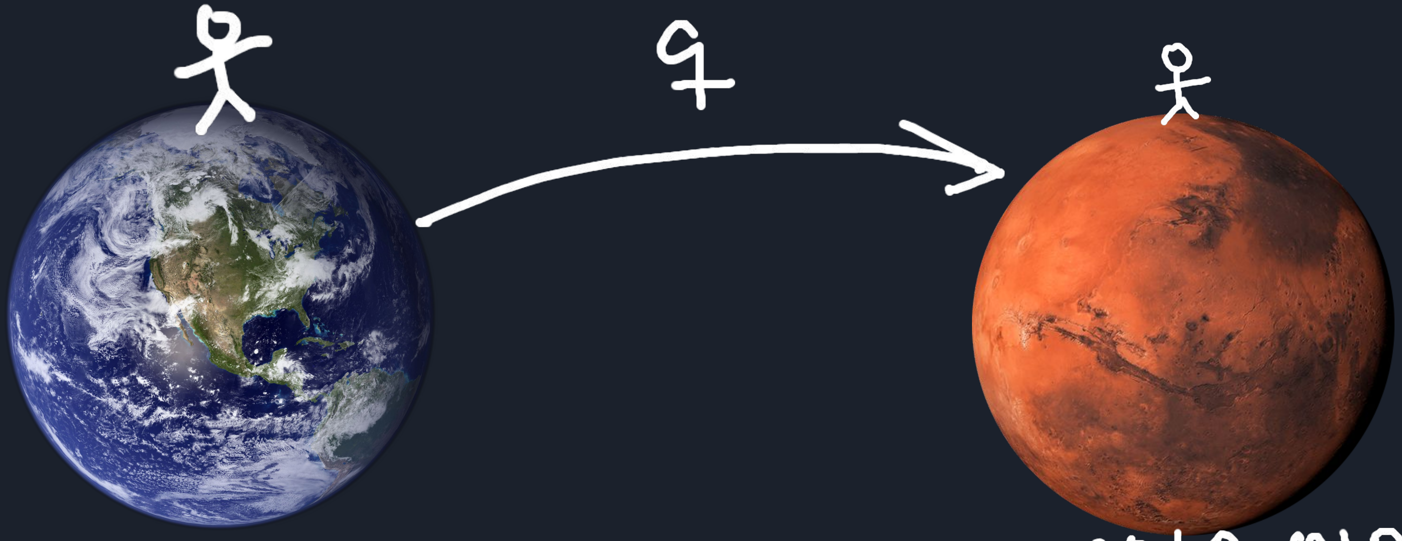
Budget:
O(log(q))



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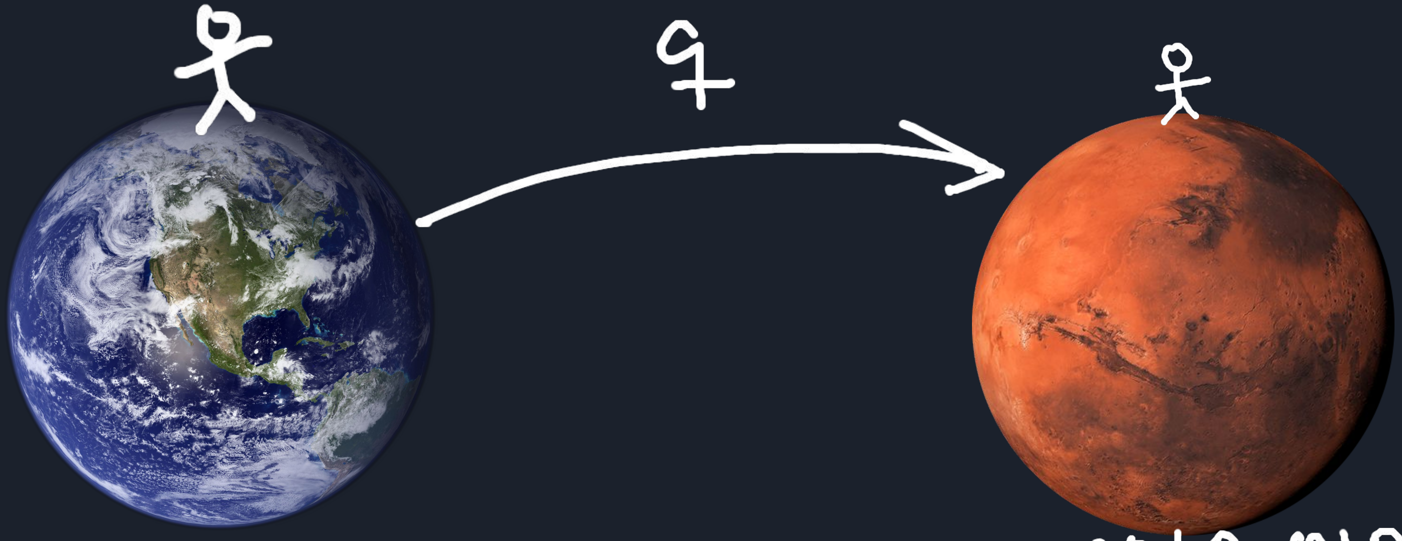
Budget:
$O(\log(n^2))$



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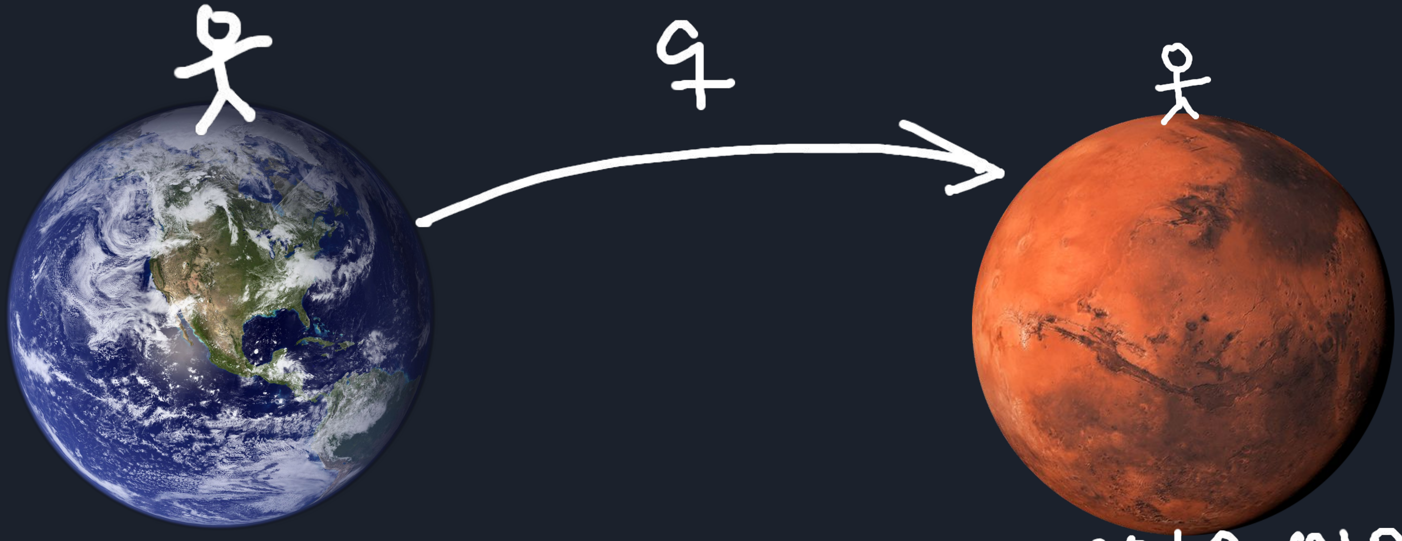

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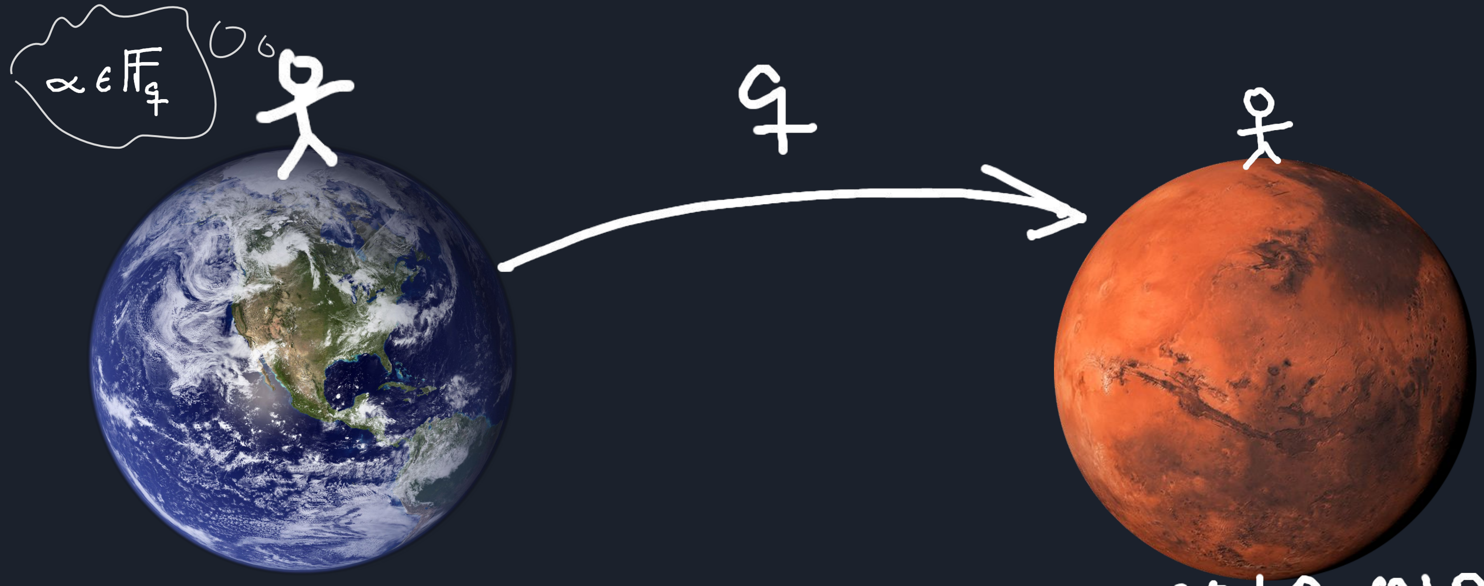

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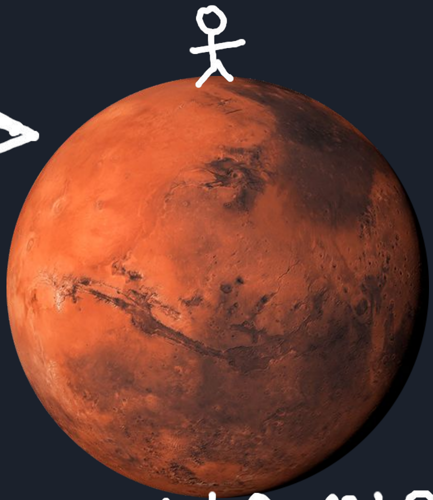
Budget:
O(log(n))
O(log(α))

q

α



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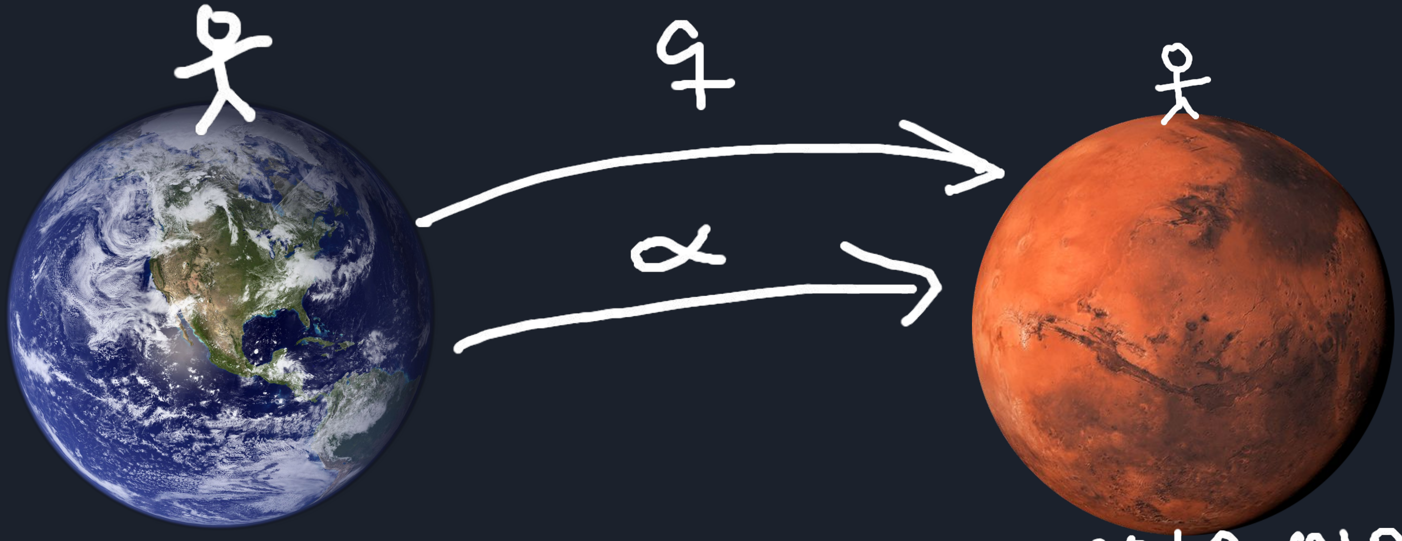

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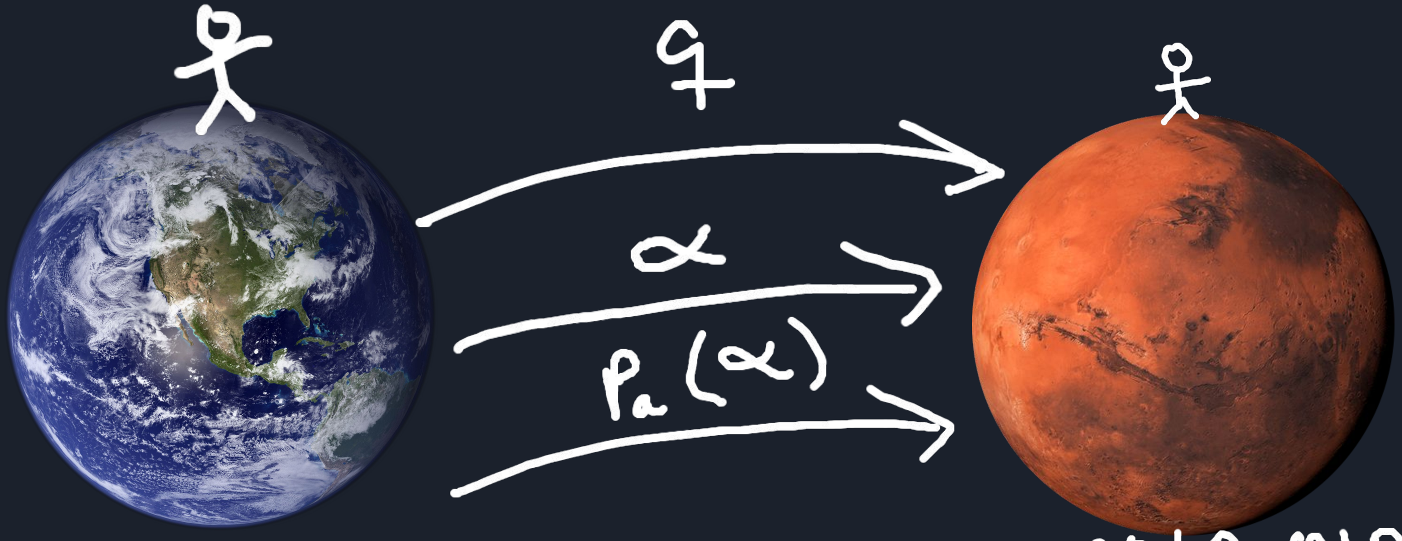

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Budget:
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O(log(n))
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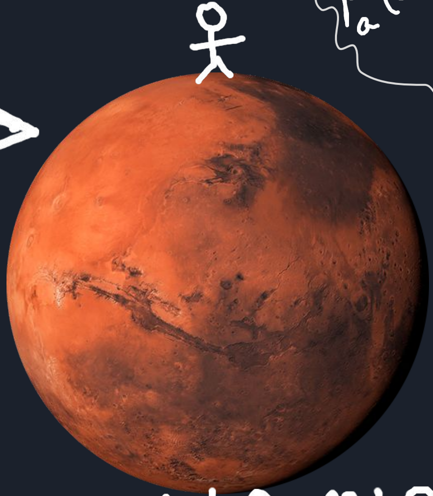
$\xrightarrow{q}$

$\xrightarrow{\alpha}$

$\xrightarrow{p_a(\alpha)}$



$p_a(\alpha) \stackrel{?}{=} p_b(\alpha)$

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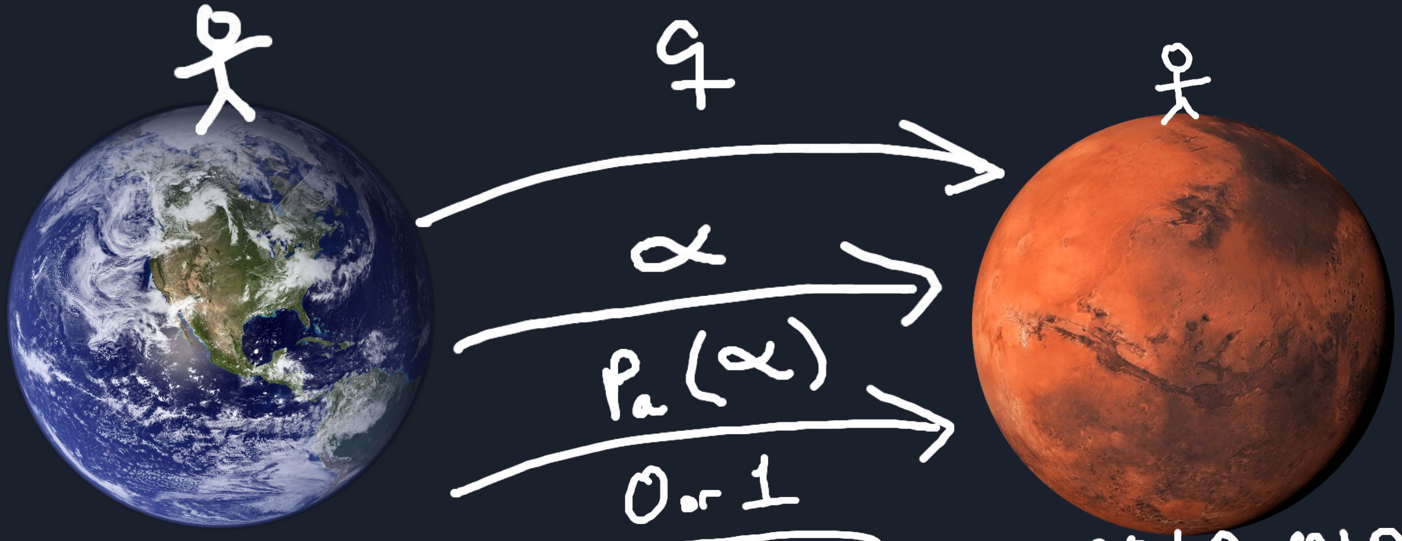
Budget:
O(log(n))
O(log(n))
O(log(n))
O(1)



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O(1)

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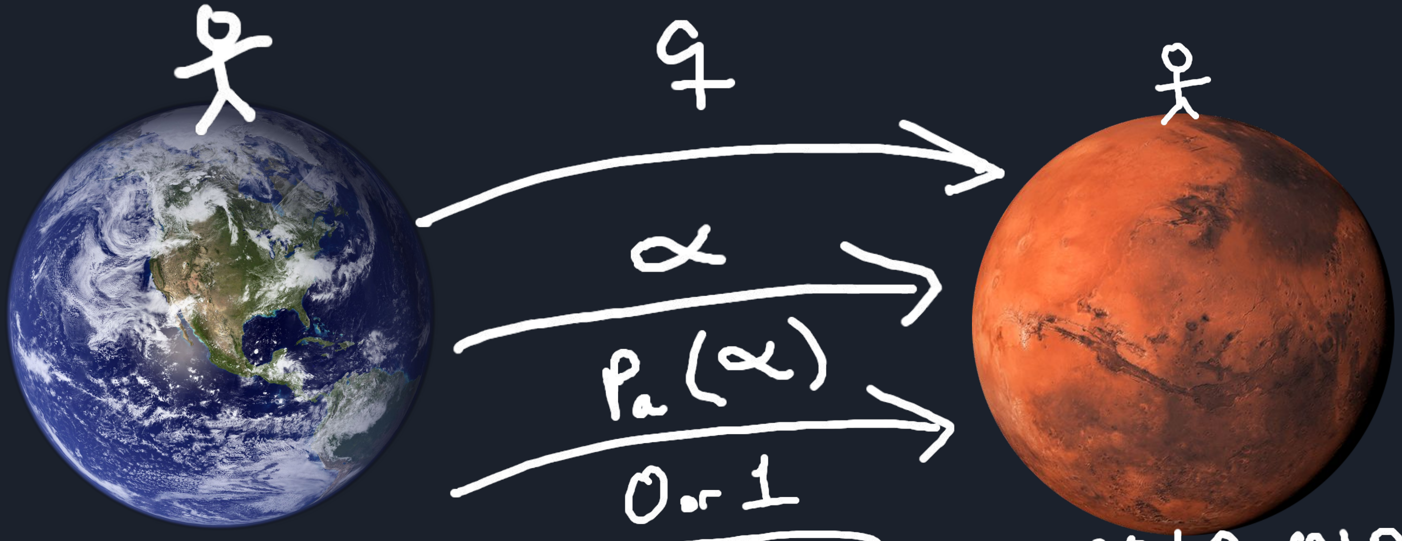

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if $a = b$, $\rho_a = \rho_b$

So $\forall \alpha .\ \rho_a \alpha = \rho_b \alpha$

So 100% of the time,

Bob tells Alyss

YES

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$\iff (\rho_a - \rho_b)(\alpha) = 0$

if $a = b$, $p_a = p_b$

So $\forall \alpha$. $p_a \alpha = p_b \alpha$

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but $\leqslant n$ α are roots of a polynomial of degree n.

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so $P_r[\text{false "yes"}]$

$= P_r[\alpha \text{ root of } P_a - P_b]$

$\leqslant \dfrac{\deg(P_a - P_b)}{q} \leqslant \dfrac{n}{n^2} = \dfrac{1}{n}$

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but $\leq n$ α are roots of a polynomial of degree n.

so $\Pr[\text{false "yes"}]$

$$= \Pr[\alpha \text{ root of } p_a - p_b]$$

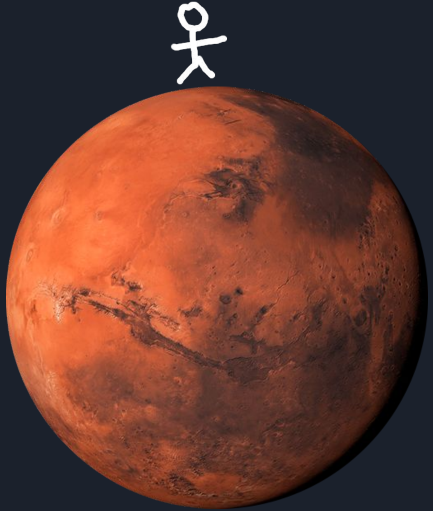
$$\leq \frac{\deg(p_a - p_b)}{q} \leq \frac{n}{n^2} = \frac{1}{n}$$

Can we do better?

Can we do better?

Yes! (If we cheat a little)

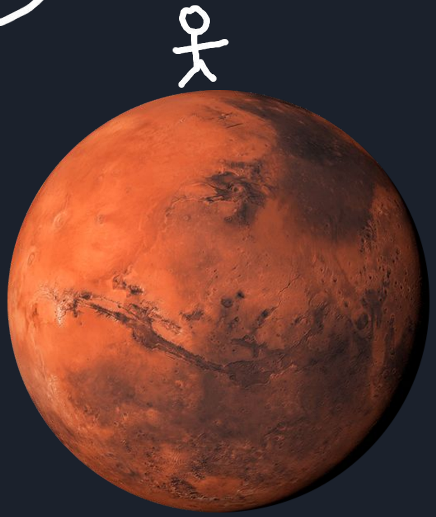
$a \in \{0,1\}^n$



$b \in \{0,1\}^n$

Random Bits In the Sky.

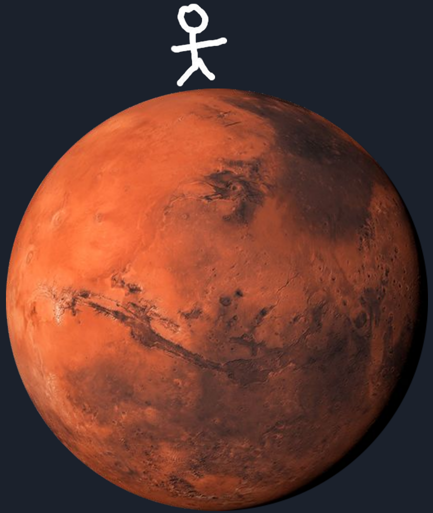
$a \in \{0,1\}^n$



$b \in \{0,1\}^n$

5,778.029364.... K

$a \in \{0,1\}^n$



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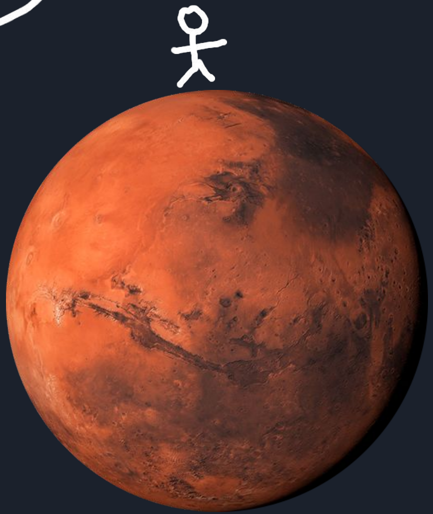

$a \in \{0,1\}^n$

$b \in \{0,1\}^n$

Theorem:

In the “public randomness” model, Alyss and Bob can solve Eq_n with probability of a false positive < 25% using…

~ Audience Participation ~

Theorem:

In the “public randomness” model, Alyss and Bob can solve Eq_n with probability of a false positive < 25% using…

3 bits of communication!

Theorem:

In the “public randomness” model, Alyss and Bob can solve Eq_n with probability of a false positive $< \epsilon$ using $O(\log(1/\epsilon))$ bits of communication

Uniform in n!

Random Bits In the Sky.

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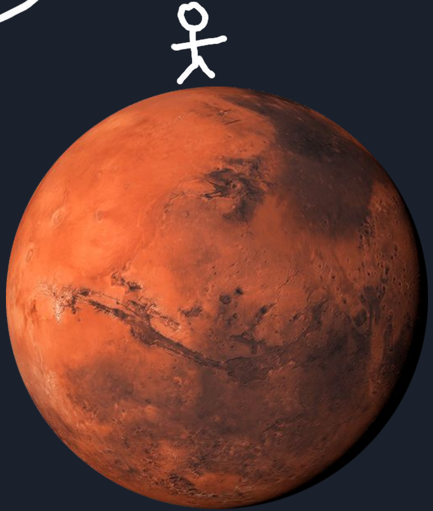
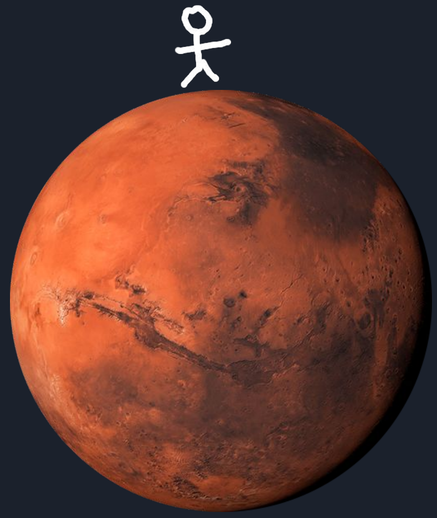


$b \in \{0,1\}^n$

$r_1 \cdots r_n \quad s_1 \cdots s_n \cdots$

$a \in \{0,1\}^n$



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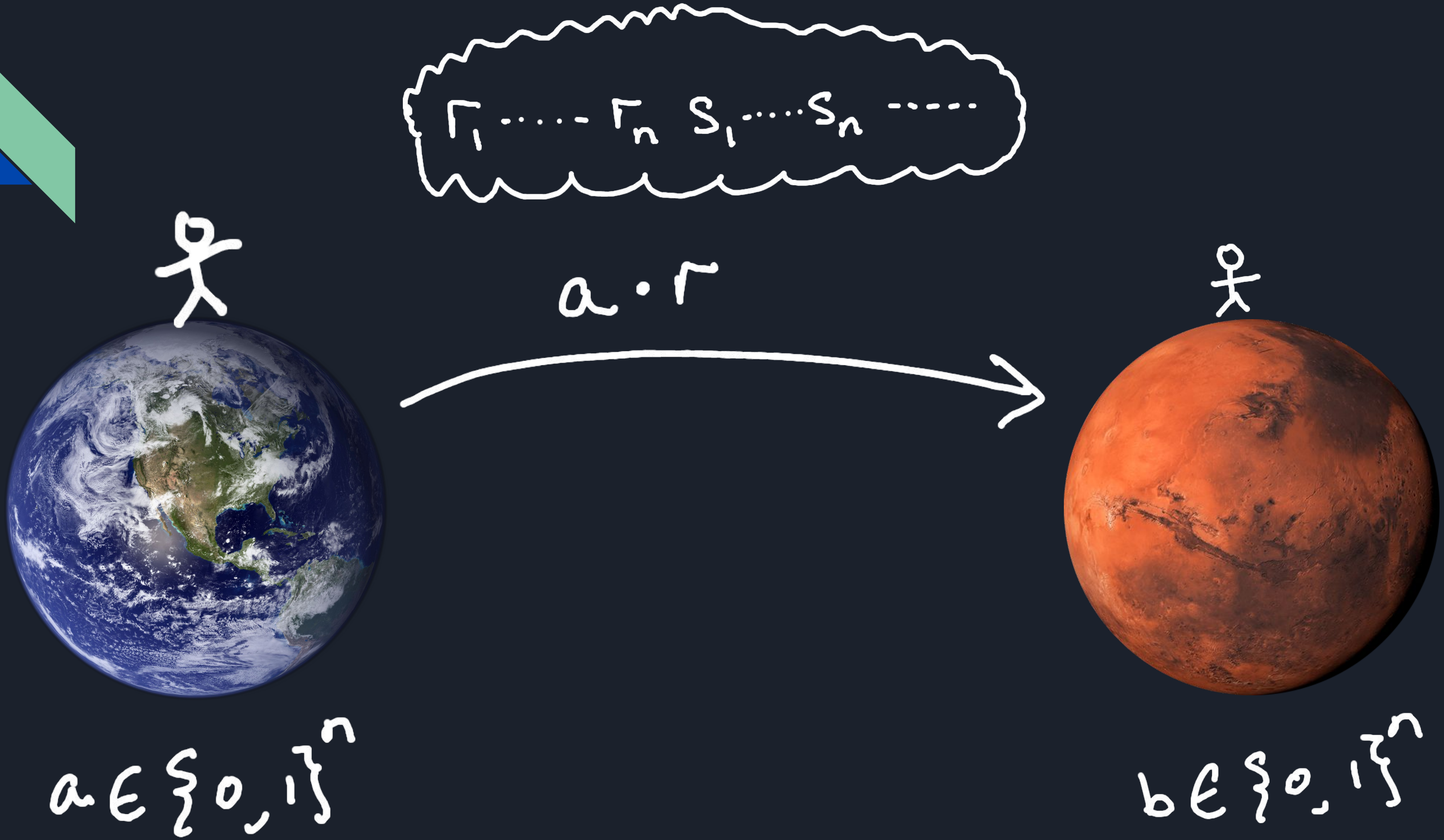
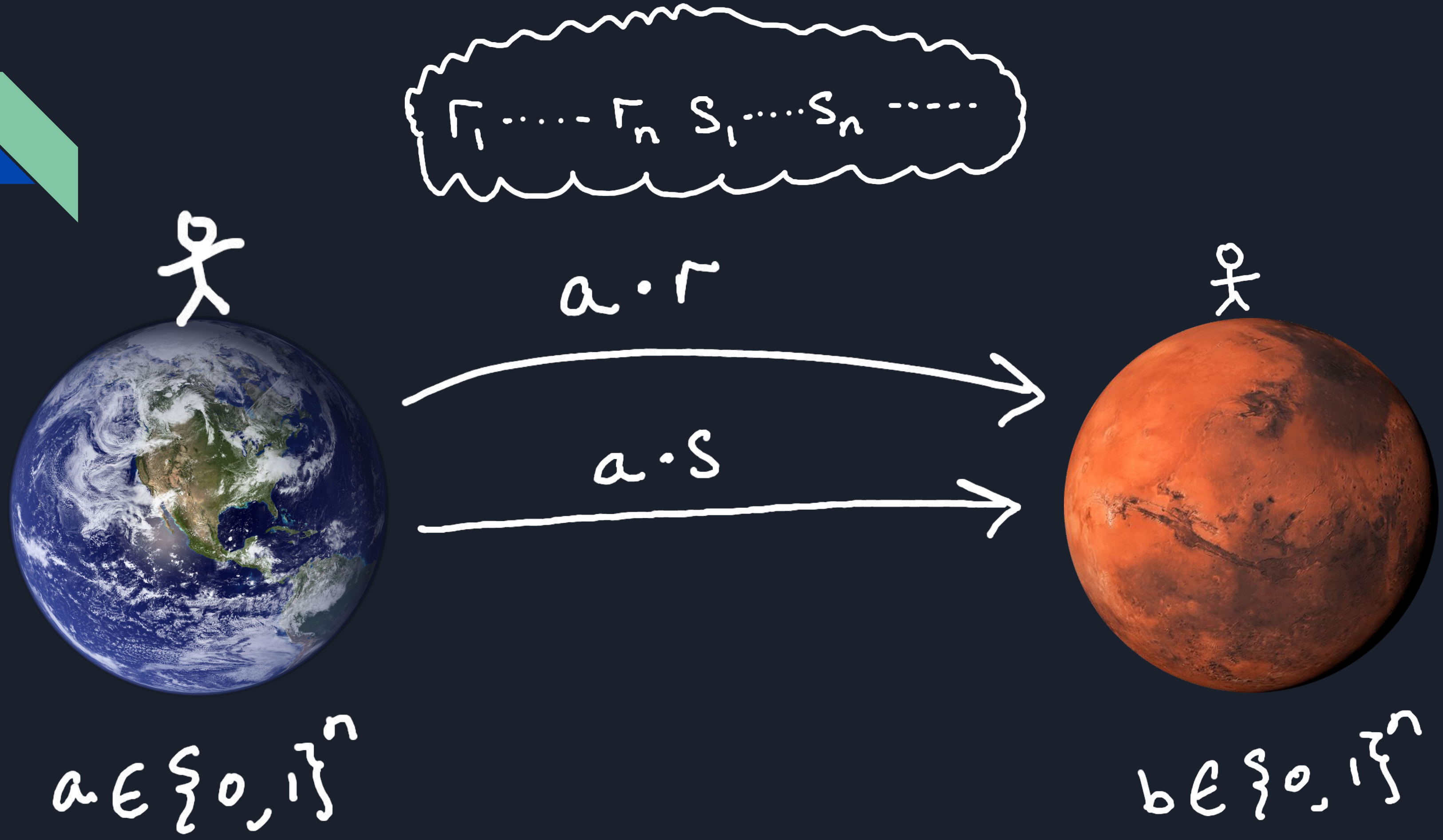
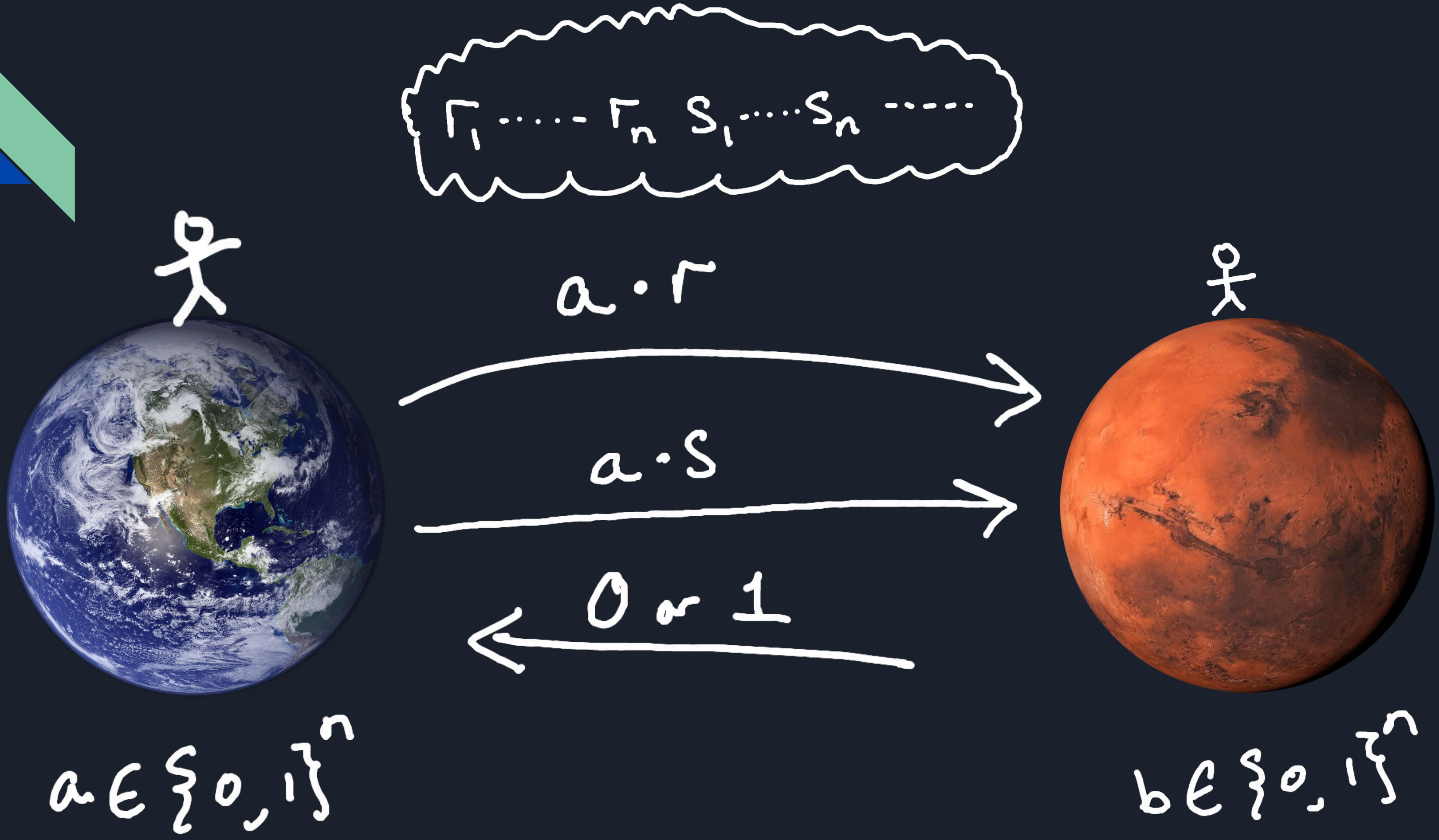


$r_1 \cdots\cdots r_n \; s_1 \cdots\cdots s_n \cdots\cdots$
$a \cdot r$
$a \in \{0,1\}^n$
$b \in \{0,1\}^n$


$r_1 \cdots\cdots r_n \; s_1 \cdots\cdots s_n$ -----
$a \cdot r$
$a \cdot s$
$a \in \{0,1\}^n$
$b \in \{0,1\}^n$


$r_1 \cdots\cdots r_n \; s_1 \cdots\cdots s_n \cdots\cdots$
$a \cdot r$
$a \cdot s$
0 or 1
$a \in \{0,1\}^n$
$b \in \{0,1\}^n$

If $a = b$, clearly $\forall r.\ a \cdot r = b \cdot r$

Otherwise, note

$$\Pr[a \cdot r = 1] = \frac{1}{2}.$$

So $\Pr[a \cdot r = b \cdot r] = \frac{1}{2}$

So $\Pr[a \cdot r = b \cdot r \ \& \ a \cdot s = b \cdot s] = \frac{1}{4}$.

As we said, though:
25% isn't special.

If you want Pr[☹] < 0.01,
then sending 7 bits
is always enough:

$$\frac{1}{2^7} = \frac{1}{128} < \frac{1}{100} = 0.01$$

In general, to get

$\Pr[\text{☹}] < \varepsilon$, we need

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$$\left(\iff \#\text{messages} > \log(1/\varepsilon)\right)$$

Ok… But how much are we cheating by?

What about private coin complexity?

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- Similarly, we write $R_{\epsilon}^{priv}(f)$ for the **R**andomized (private) communication complexity
- (In both, the subscript ϵ indicates the tolerance for errors)

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Newman’s Theorem says we will never do worse than this.

For more information, see Ryan O’Donnell’s “CS Theory Toolkit” on Youtube.

I’ll link the playlist on my blog post for this talk at grossack.site

Thank You! ^_^